Efficient Fuzzy Extraction of PUF-Induced Secrets: Theory and Applications

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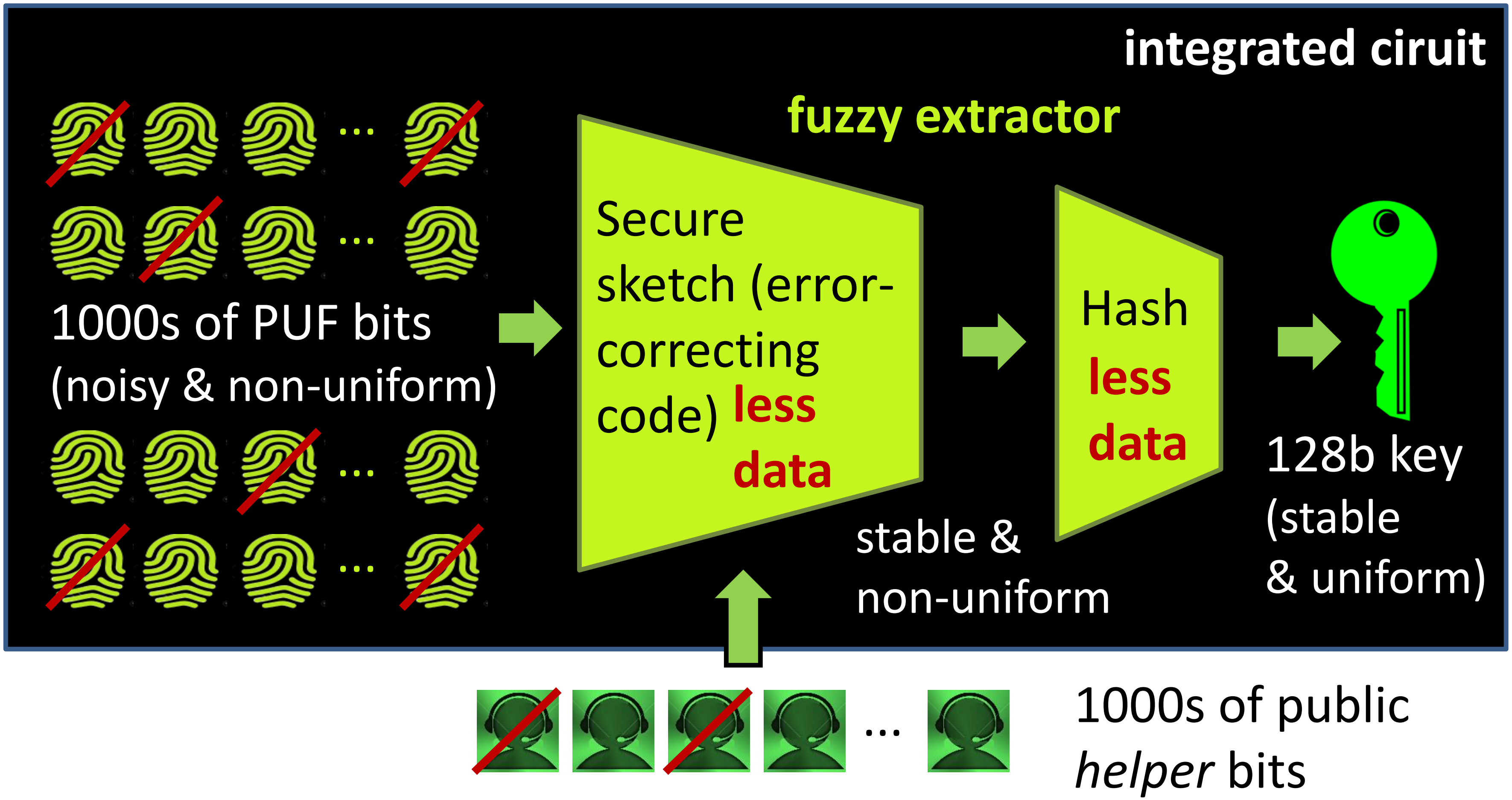
PUF-Based Key Generation

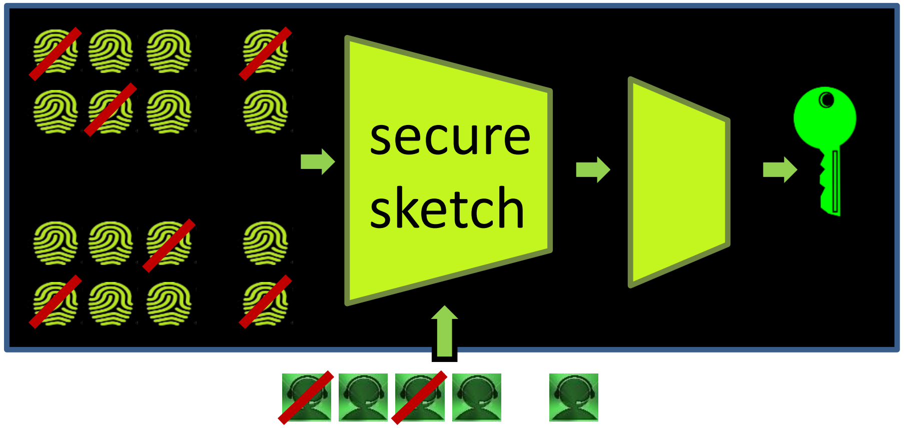
Can we make the default architecture more efficient?



Presentation Outline

(1) Preliminaries: PUF and secure sketch

(2) **THEORY**: tighter bounds on the secure sketch min-entropy loss



sorry, paper only, not here

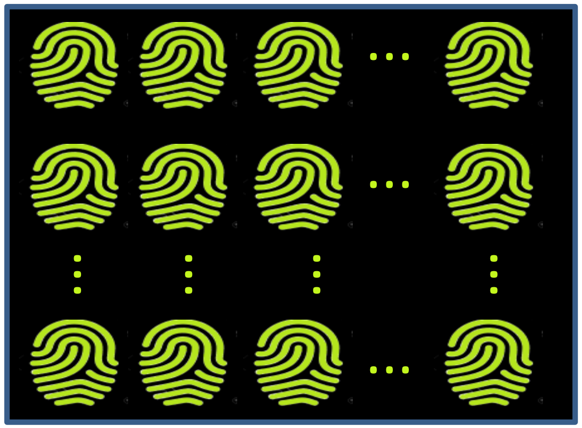
(3) **APPLICATIONS**: *Focus of this talk*

- Reduction in implementation footprint
- Debunk security proof of *reverse fuzzy extractor*
- Proper motivation for debiasing schemes

} 2 CHES 2015 papers

Preliminaries: Array-Based PUFs

Array of identical cells, each producing 1 device-unique bit



* SRAM, DRAM, DFF PUFs (memory-based)
* 1 RO-based PUF
* Coating PUF
* ~~Arbiter PUF and variations~~

issue 1: noisiness

BER 1% - 20% w.r.t. a reference response

issue 2: non-uniformities more 1 than 0, or vice versa

bias

1	0	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1

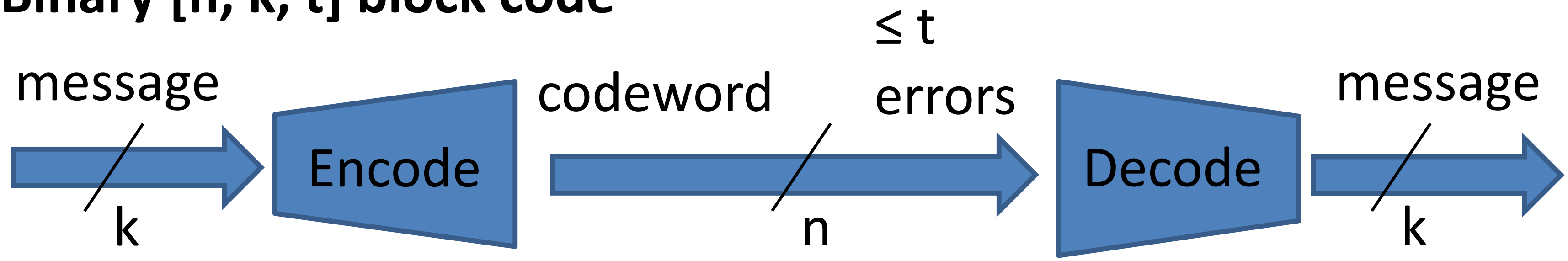
spatial correlations

1	1	1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	1

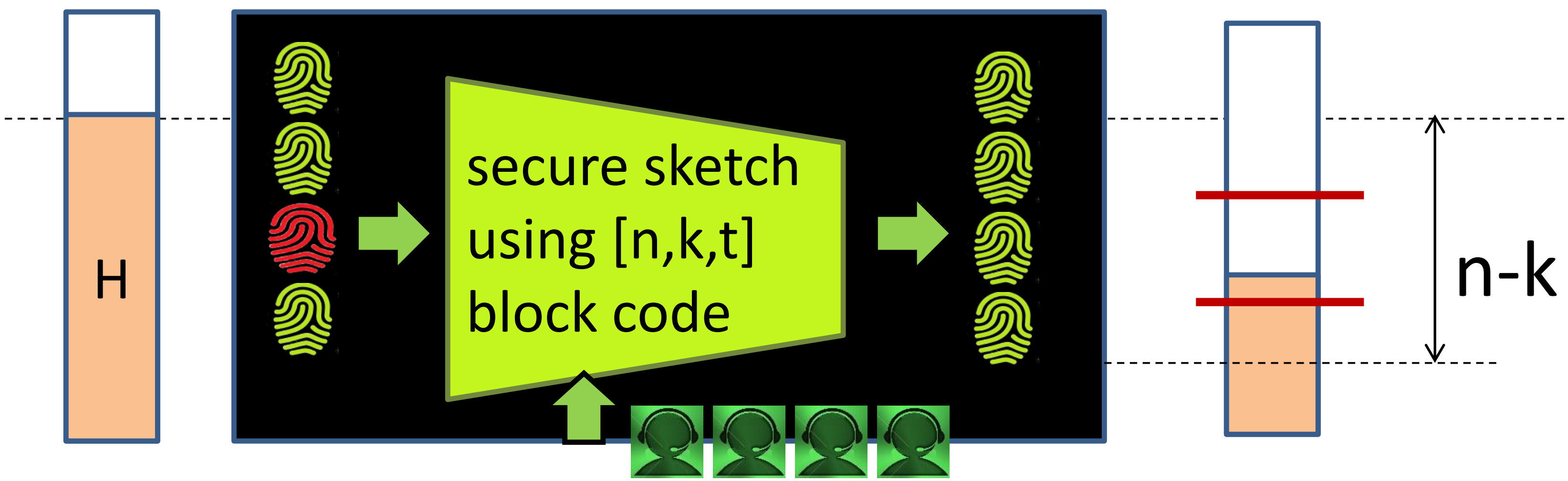
neighboring cells influence each other

Preliminaries: Secure Sketch

Binary [n, k, t] block code



System providers use (**n-k**) **upper bound** on the min-entropy loss



tighter upper/lower bounds (enclosing true value, easy-to-evaluate)

Related work: defeat (*n-k*) bound

- **New research direction**

[Delvaux et al., IEEE TCAD 2014] → [Maes et al., CHES 2015]

so far only repetition codes and i.i.d. PUF bits (bias)

1	0	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	0	1	1

Pr(x(i) = 1) = b with b ∈ [0,1]

- We considerably **extend the scope** on two fronts

<u>1) Large complex codes</u>: BCH, RM, concatenations, ...

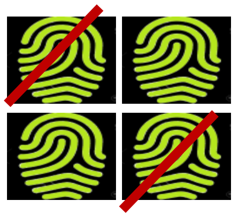
<u>2) Various PUF distributions</u>: bias, spatial correlations, ...

1	1	1	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1

Pr(x(i) = x(i+1)) = c with c ∈ [0,1]

Application: reduce implementation footprint

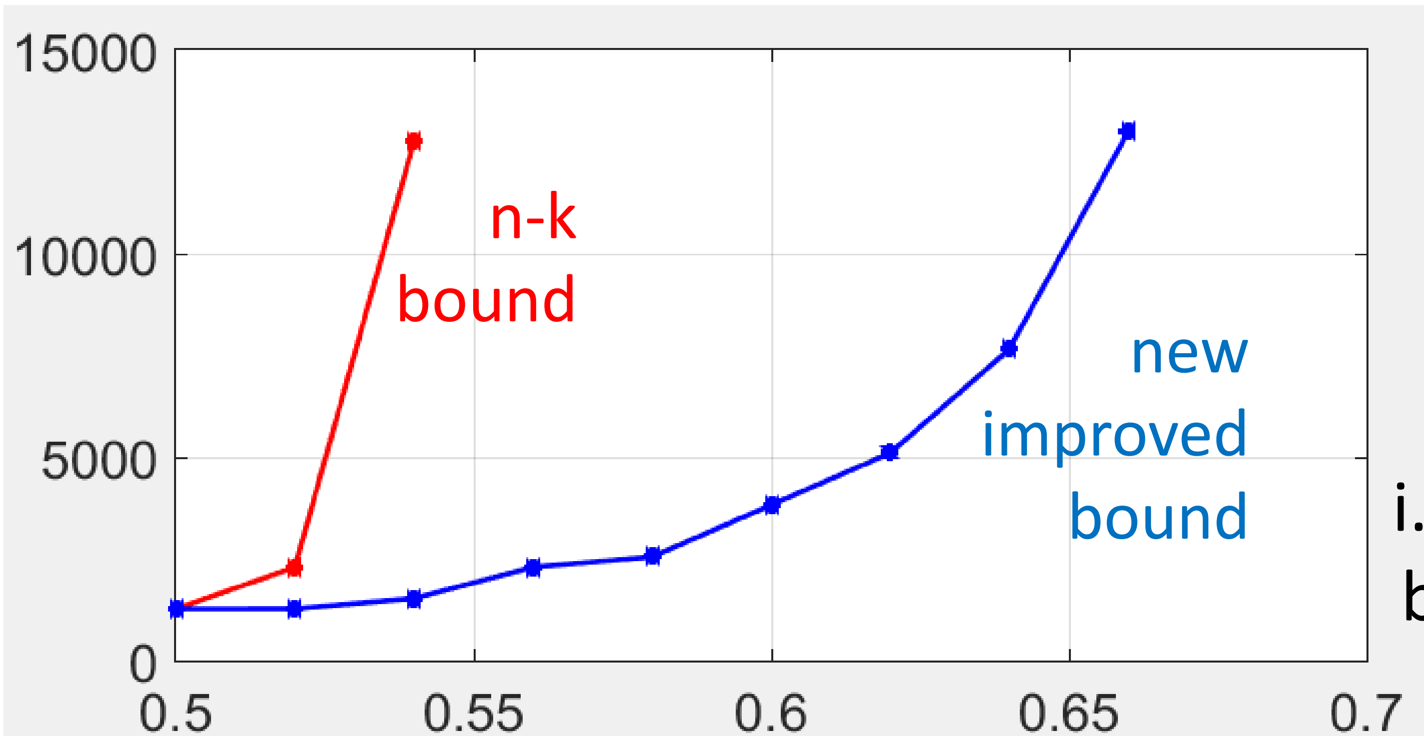
Specs: 128-bit key, BCH+REP code, Pr(error) = 0.1, Pr(fail) ≤ 1E-6



PUF bits

(also less helper bits)

(similar trend for spatially correlated distribution)



i.i.d. bias *b*

1	1	1	1	1
0	0	0	0	0

1	1	1	1	1
1	0	0	0	0

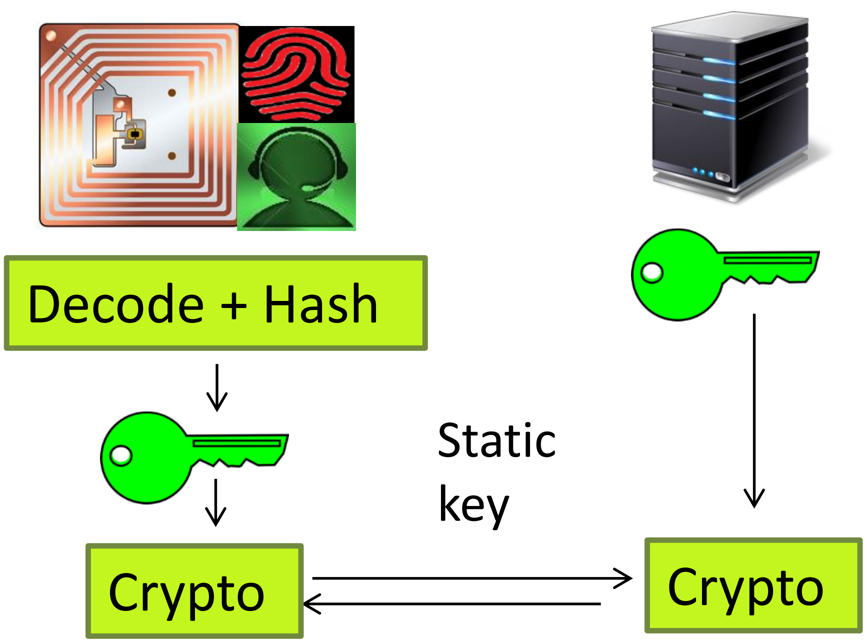
1	1	1	1	1
1	1	0	0	0

Application: Reverse fuzzy extractor (1/2)

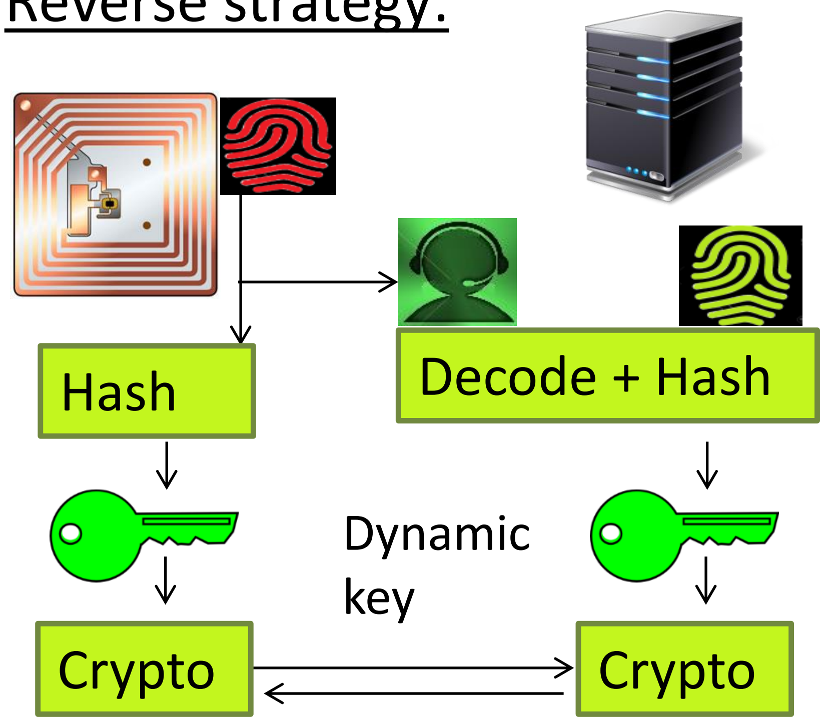
Technique to reduce footprint of PUF-based protocols:

[Van Herrewege et al., FC 2012] → [Maes et al., PhD 2012] → [Aysu et al., CHES 2015, DATE 2016]

Conventional strategy:



Reverse strategy:



Application: Reverse fuzzy extractor (2/2)

Claim: repeated helper data exposure does <u>not</u> result in additional min-entropy loss

~~**Proof:** from [Boyen, ACM CCS 2004]~~ flawed transfer

implicit exposure of individual bit error rates is overlooked

Intuition of unanticipated entropy loss: for biased PUF

PUF error statistics

e.g., [Maes, CHES 2013]

1	0	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1

1	70%	E[BER] ≈ 9%	overall E[BER] ≈ 10%
0	30%	E[BER] ≈ 13%	

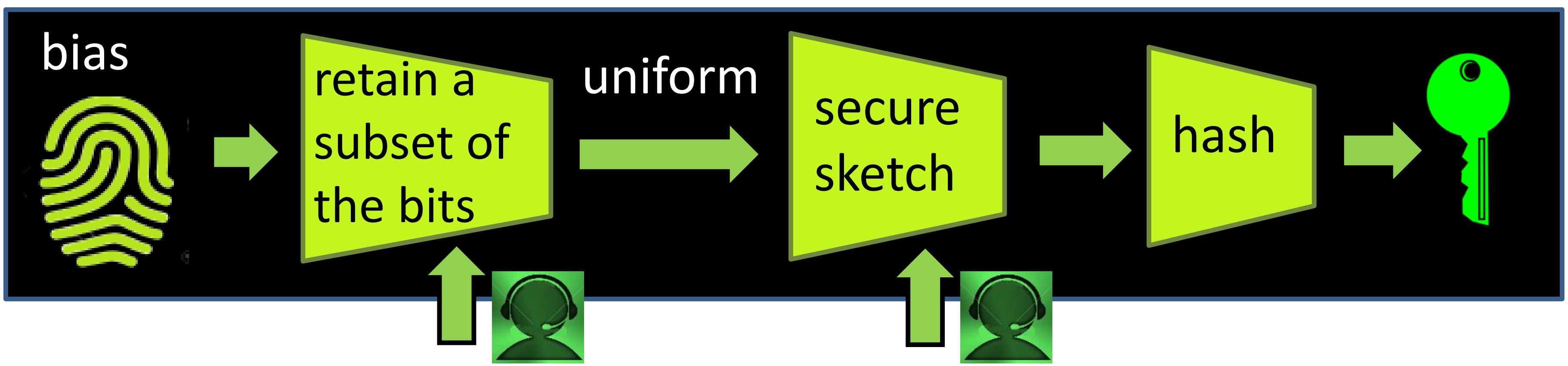
practice: conservative (n-k) bound acts as counterweight

Application: motivation for debiasing schemes

IBS: [Yu et al., IEEE 2010] → [Hiller et al., HOST 2012]

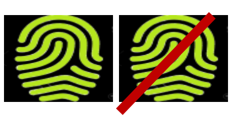
von Neumann: [van der Leest et al., ACM STC 2010] → [Van Herrewege, PhD 2015] → [Maes et al., CHES 2015]

Conjecture that a stand-alone sketch cannot handle bias (which is correct in case the n-k bound is applied)



Application: motivation for debiasing schemes

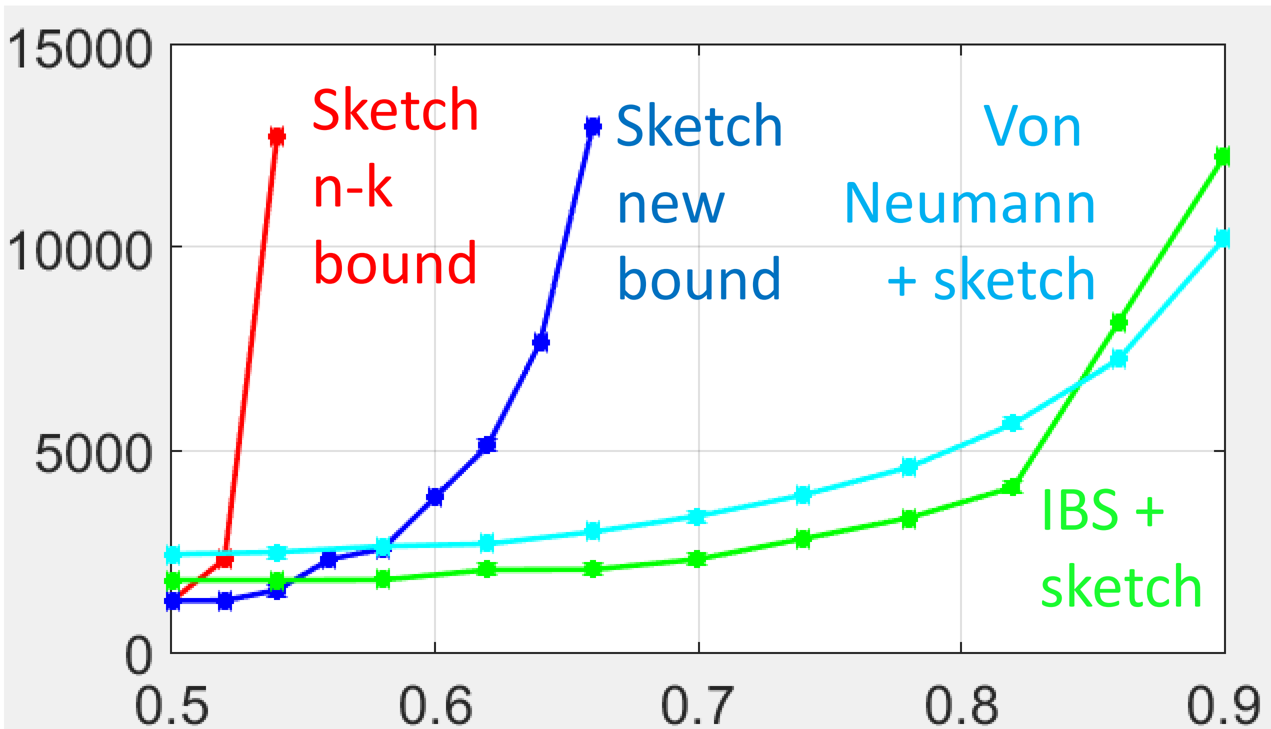
Also in favor of a stand-alone sketch:
1) Debiasing is not for free (circuitry and helper data)
2) Applies to more distributions (other than i.i.d. bias)



PUF bits

new bound is competitive

need for debiasing schemes (or better: PUF redesign)



bias b

Summary

THEORY: **tight bounds** on the secure sketch min-entropy loss for array-based PUFs (new research direction, open for further exploration & improvements)

APPLICATION: reduce fuzzy extrator **implementation footprint**, compared to (n-k) bound

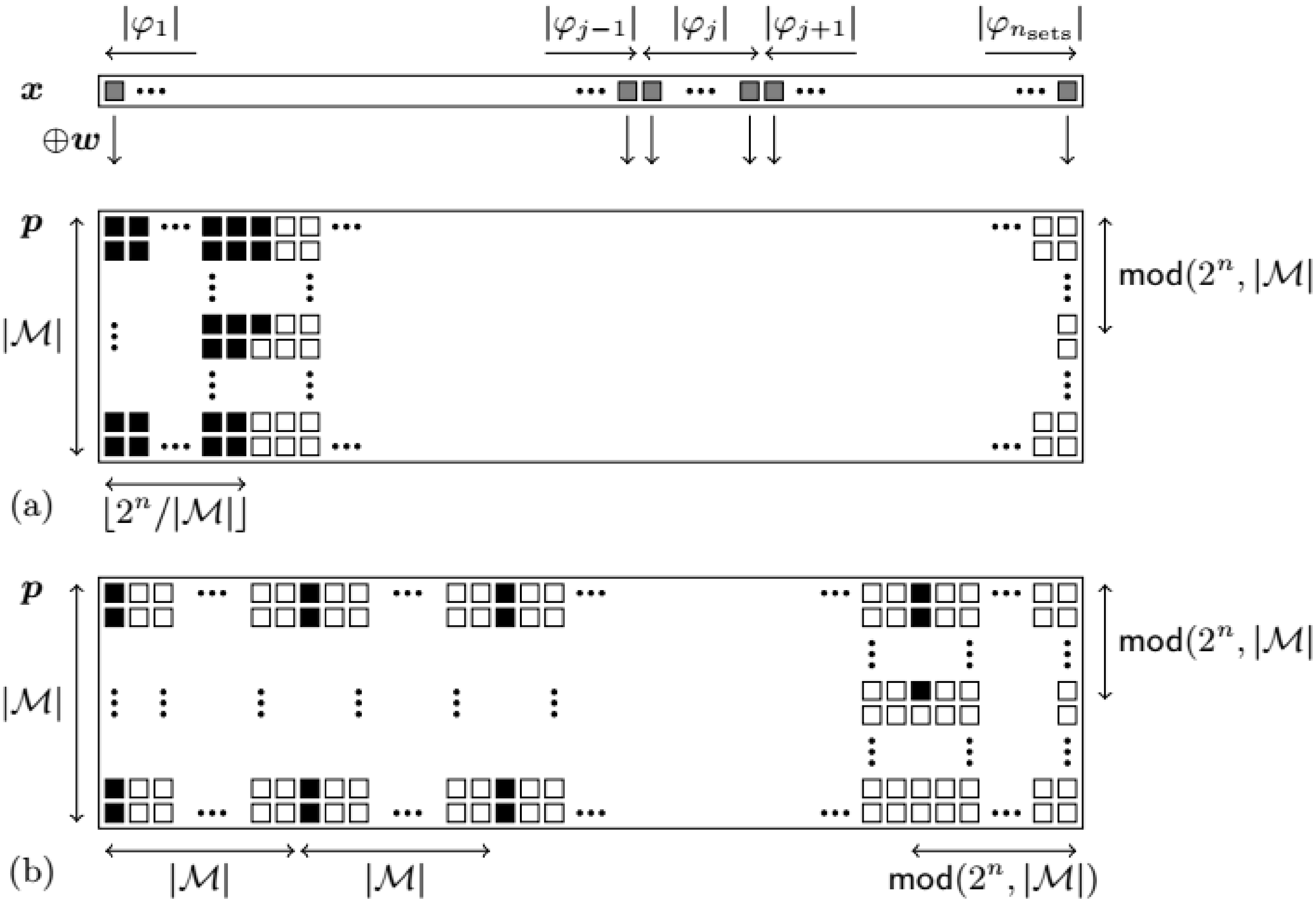
APPLICATION: debunk security proof of **reverse fuzzy extractor** (open for repairs)

APPLICATION: motivate the need for **debiasing schemes** (although low-bias PUFs can do without)

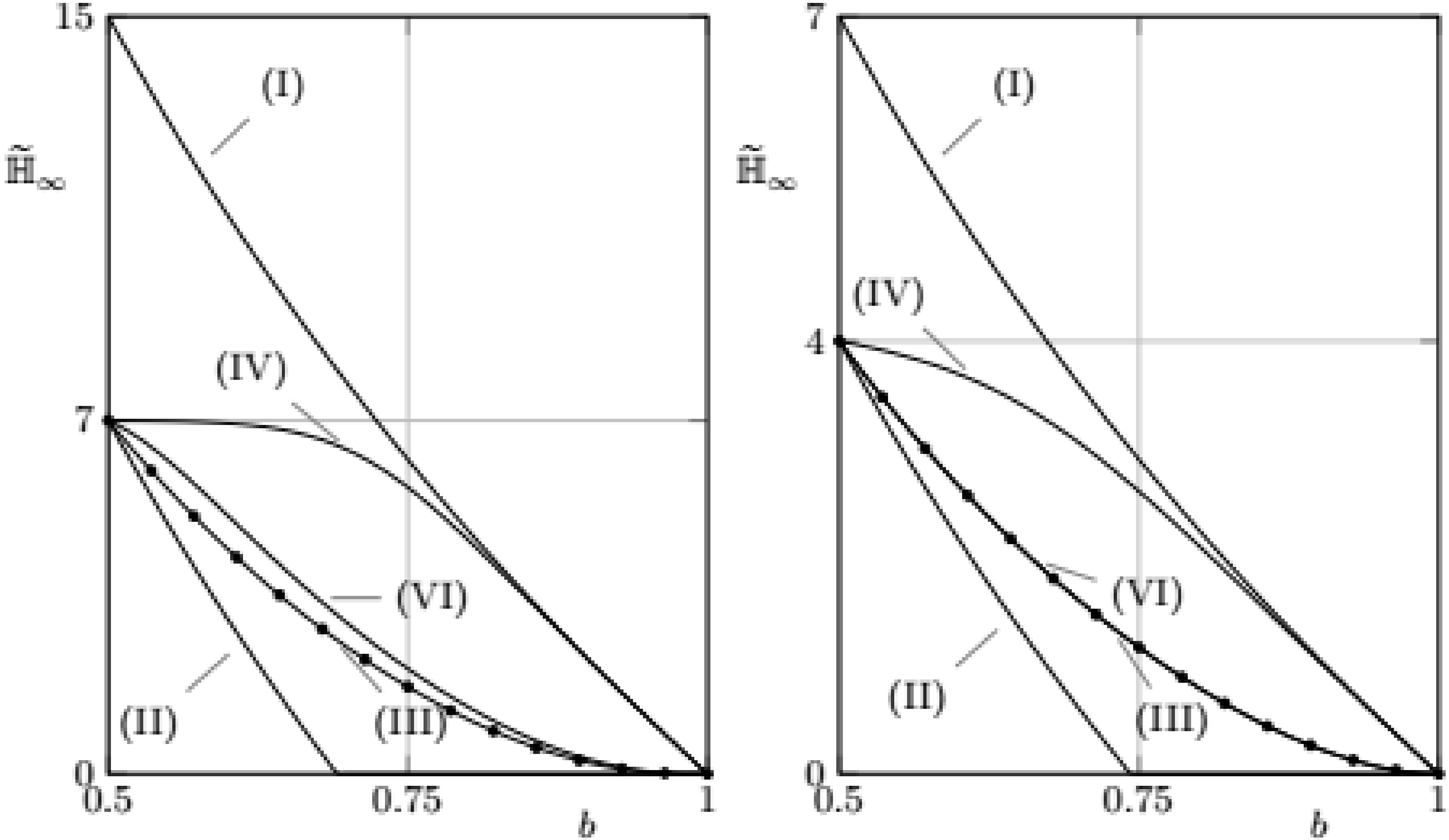
Thank you!

Appendix

All figures and tables

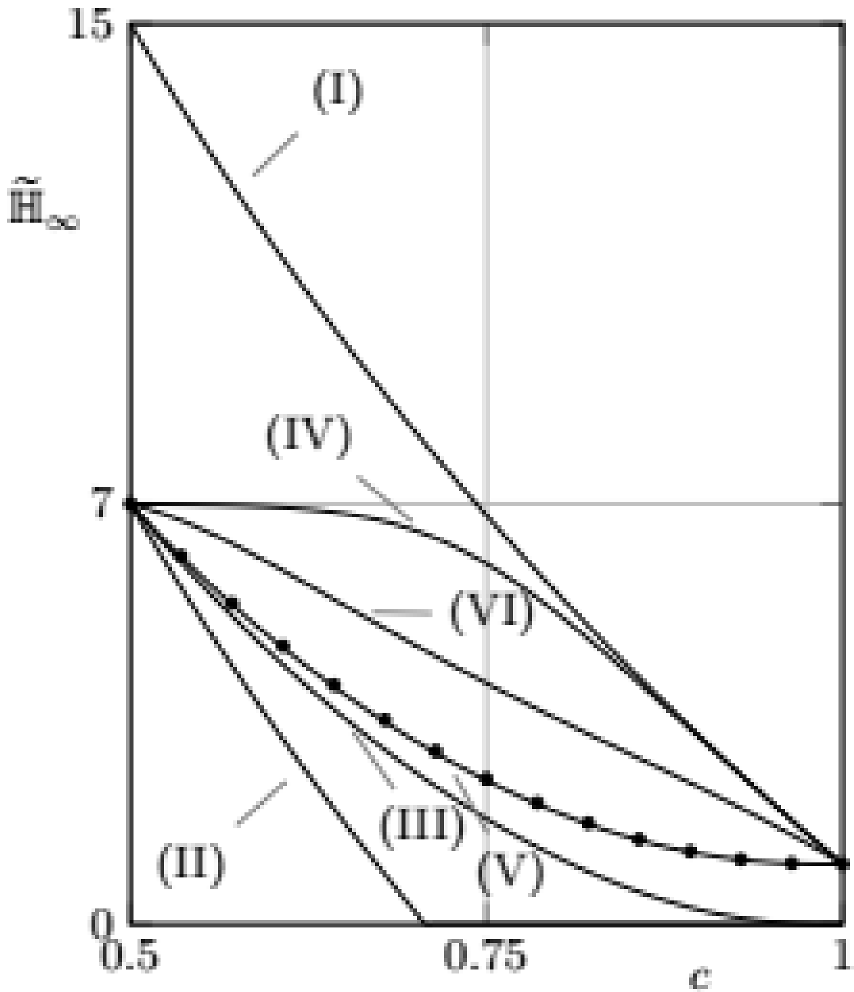


$|\varphi_1|$
$|\varphi_{j-1}|$
$|\varphi_j|$
$|\varphi_{j+1}|$
$|\varphi_{n_{\text{sets}}}|$
$\boldsymbol{x}$
$\oplus \boldsymbol{w}$
$\boldsymbol{p}$
$|\mathcal{M}|$
$\text{mod}(2^n, |\mathcal{M}|)$
(a)
$\lfloor 2^n/|\mathcal{M}| \rfloor$
$\boldsymbol{p}$
$|\mathcal{M}|$
$\text{mod}(2^n, |\mathcal{M}|)$
(b)
$|\mathcal{M}|$
$|\mathcal{M}|$
$\text{mod}(2^n, |\mathcal{M}|)$

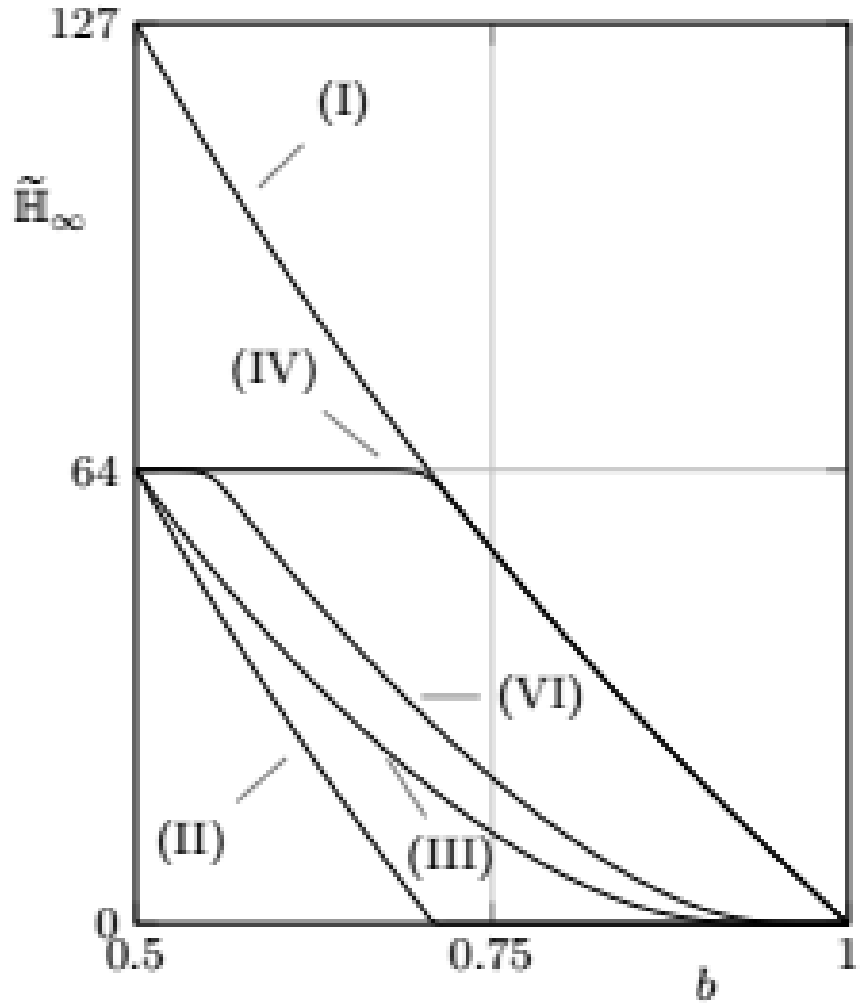


(a) Bias; $[n = 15, k = 7, d = 5]$.

(b) Bias; $[n = 7, k = 4, d = 3]$.

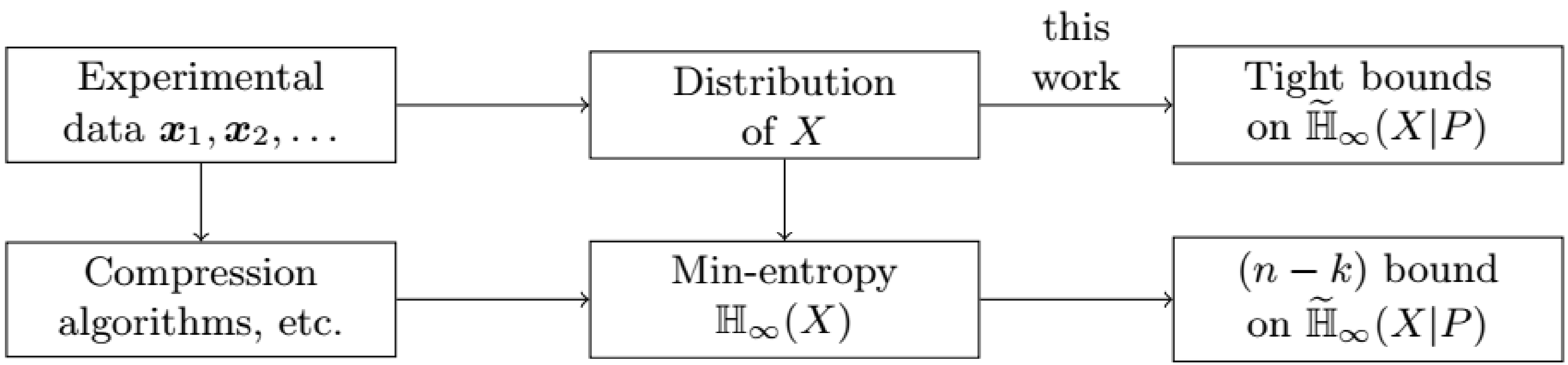


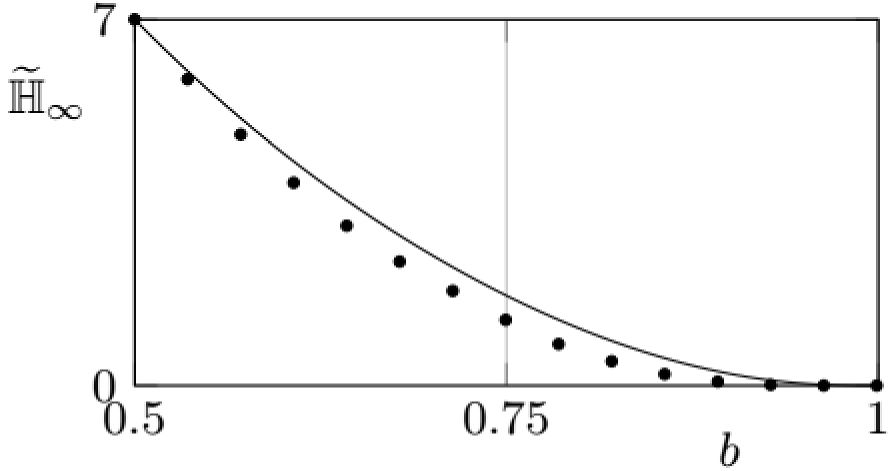
(c) Correlation; $[n = 15, k = 7, d = 5]$.



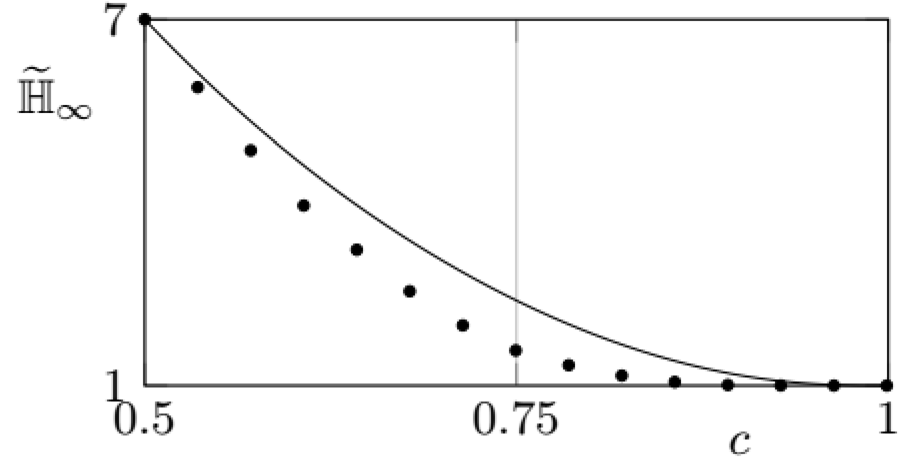
(d) Bias; $[n = 127, k = 64, d = 21]$.

	b	$\mathbb{E}[P_{\text{error}}]$	$z \times [n_2, k_2, d_2] \circ [n_1, k_1, d_1]$	$\widetilde{\mathbb{H}}_\infty(X\|P)$	PUF size n	$\mathbb{E}[P_{\text{fail}}]$
$n-k$ bound	0.50	$\approx 10.0\%$	$2 \times [5,1,5] \circ [127,64,21]$	128	1270	$\approx 3.26\text{E}{-8}$
	0.52	$\approx 10.0\%$	$3 \times [3,1,3] \circ [255,87,53]$	≈ 131.1	2295	$\approx 1.44\text{E}{-8}$
	0.54	$\approx 9.96\%$	$10 \times [5,1,5] \circ [255,155,27]$	≈ 134.4	12750	$\approx 5.56\text{E}{-7}$
	0.56	$\approx 9.90\%$	No code within the search space satisfies the constraints.			
new bound	0.50	$\approx 10.0\%$	$2 \times [5,1,5] \circ [127,64,21]$	128	1270	$\approx 3.26\text{E}{-8}$
	0.52	$\approx 10.0\%$	$1 \times [5,1,5] \circ [255,163,25]$	≈ 134.3	1275	$\approx 4.27\text{E}{-7}$
	0.54	$\approx 9.96\%$	$2 \times [3,1,3] \circ [255,99,47]$	≈ 132.5	1530	$\approx 5.35\text{E}{-7}$
	0.56	$\approx 9.90\%$	$3 \times [3,1,3] \circ [255,87,53]$	≈ 131.3	2295	$\approx 9.90\text{E}{-9}$
	0.58	$\approx 9.81\%$	$2 \times [5,1,5] \circ [255,163,25]$	≈ 130.0	2550	$\approx 4.85\text{E}{-7}$
	0.60	$\approx 9.71\%$	$3 \times [5,1,5] \circ [255,155,27]$	≈ 129.5	3825	$\approx 6.96\text{E}{-8}$
	0.62	$\approx 9.58\%$	$4 \times [5,1,5] \circ [255,163,25]$	≈ 130.4	5100	$\approx 4.42\text{E}{-7}$
	0.64	$\approx 9.42\%$	$10 \times [3,1,3] \circ [255,99,47]$	≈ 132.8	7650	$\approx 3.87\text{E}{-7}$
	0.66	$\approx 9.24\%$	$17 \times [3,1,3] \circ [255,99,47]$	≈ 129.7	13005	$\approx 3.28\text{E}{-7}$

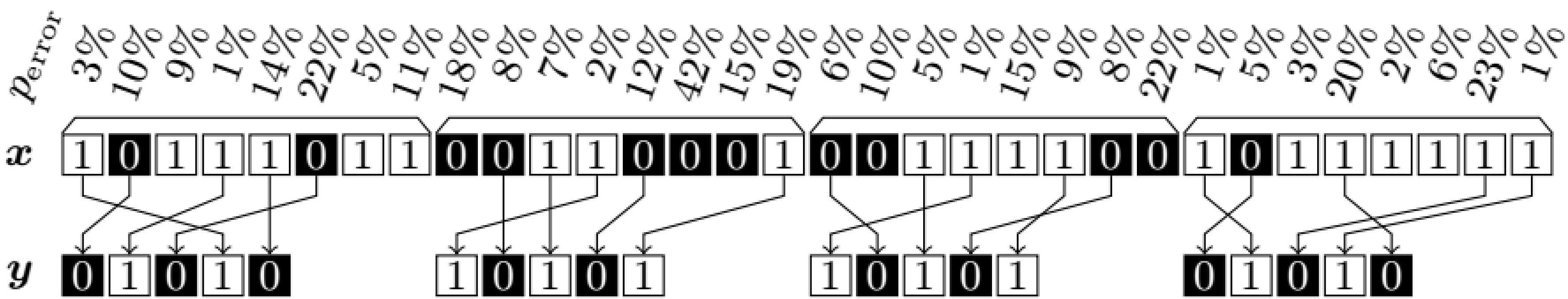

Experimental data $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$
Distribution of X
this work
Tight bounds on $\widetilde{\mathbb{H}}_\infty(X|P)$
Compression algorithms, etc.
Min-entropy $\mathbb{H}_\infty(X)$
$(n-k)$ bound on $\widetilde{\mathbb{H}}_\infty(X|P)$



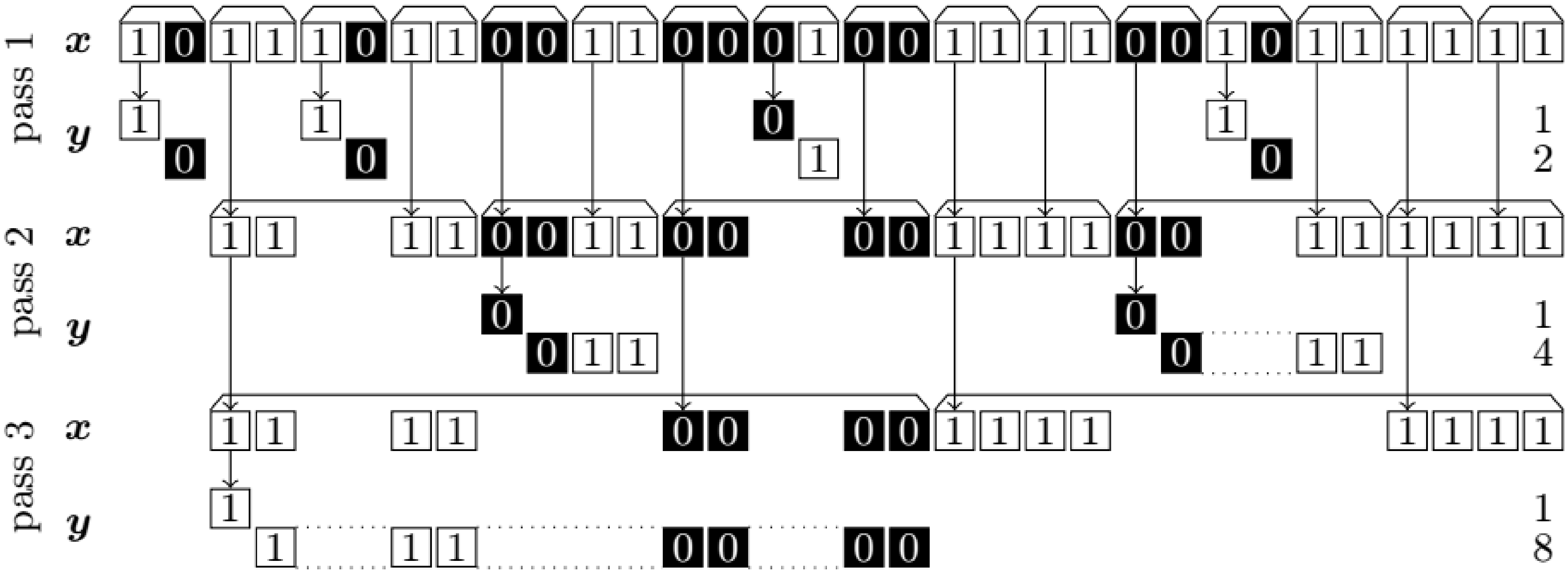
(a) Bias; $[n = 15, k = 7, d = 5]$.

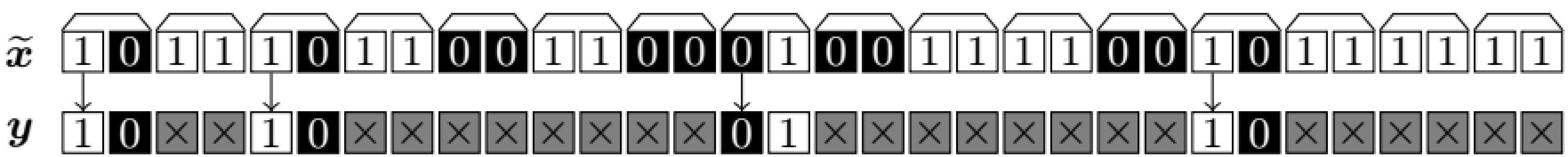


(a) Correlation; $[n = 15, k = 7, d = 5]$.



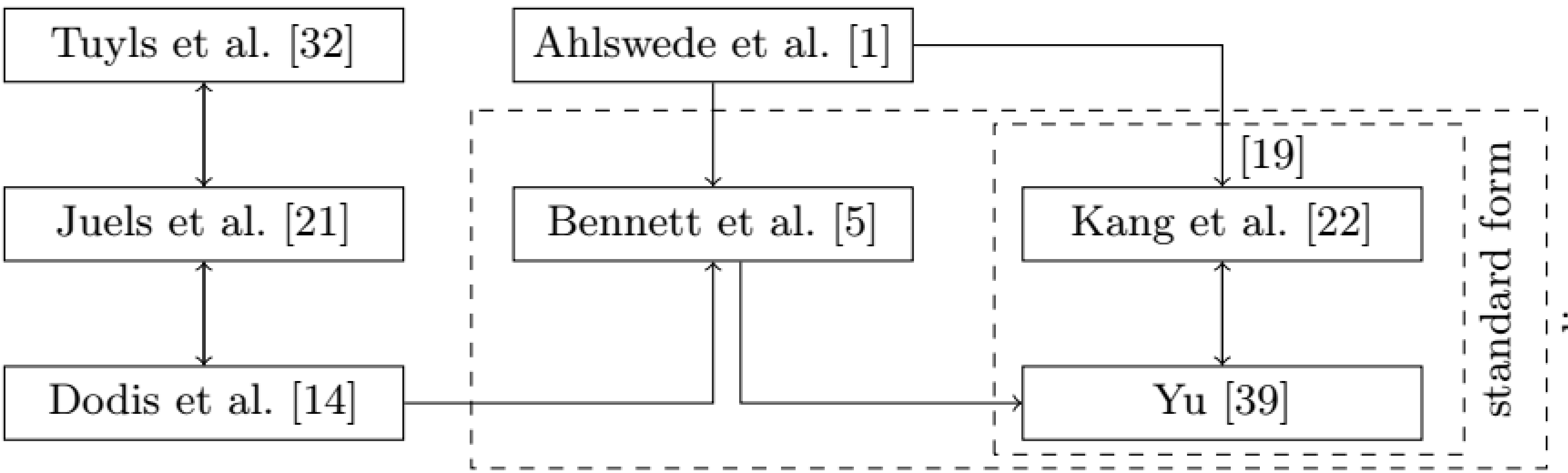
p_{error}
3% 10% 9% 1% 14% 22% 5% 11% 18% 8% 7% 2% 12% 42% 15% 19% 6% 10% 5% 1% 15% 9% 8% 22% 1% 5% 3% 20% 2% 6% 23% 1%
$\boldsymbol{x}$
1 0 1 1 1 0 1 1 0 0 1 1 0 0 0 1 0 0 1 1 1 1 0 0 1 0 1 1 1 1 1 1
$\boldsymbol{y}$
0 1 0 1 0 1 0 1 0 1 1 0 1 0 1 0 1 0 1 0


pass 1
pass 2
pass 3
x
y
1
2
1
4
1
8


x̃
y

	b	$\mathbb{E}[P_{\text{error}}]$	$\mathbb{E}[P_{\text{error}}\|0]$	$\mathbb{E}[P_{\text{error}}\|1]$	Parameters	Retention	$z \times [n_2, k_2, d_2] \circ [n_1, k_1, d_1]$	$\widetilde{\mathbb{H}}_\infty(X\|P)$	PUF size n	$\mathbb{E}[p_{\text{fail},\mathcal{C}_2}]$	$\mathbb{E}[P_{\text{fail}}]$
Generalized IBS	0.50	$\approx 10.0\%$	$\approx 10.0\%$	$\approx 10.0\%$	$n_{\text{index}} = 7$	$\approx 71.4\%$	$2 \times [5,1,5] \circ [127,64,21]$	128	1778	$\approx 1.01\text{E}{-2}$	$\approx 1.70\text{E}{-7}$
	0.54	$\approx 9.96\%$	$\approx 10.6\%$	$\approx 9.40\%$	$n_{\text{index}} = 7$	$\approx 71.4\%$	$2 \times [5,1,5] \circ [127,64,21]$	128	1778	$\approx 1.12\text{E}{-2}$	$\approx 4.57\text{E}{-7}$
	0.58	$\approx 9.81\%$	$\approx 11.2\%$	$\approx 8.79\%$	$n_{\text{index}} = 7$	$\approx 71.4\%$	$1 \times [5,1,5] \circ [255,131,37]$	131	1785	$\approx 1.41\text{E}{-2}$	$\approx 6.46\text{E}{-9}$
	0.62	$\approx 9.58\%$	$\approx 11.8\%$	$\approx 8.18\%$	$n_{\text{index}} = 8$	62.5%	$2 \times [5,1,5] \circ [127,64,21]$	128	2032	$\approx 1.17\text{E}{-2}$	$\approx 7.09\text{E}{-7}$
	0.66	$\approx 9.24\%$	$\approx 12.5\%$	$\approx 7.56\%$	$n_{\text{index}} = 8$	62.5%	$1 \times [5,1,5] \circ [255,131,37]$	131	2040	$\approx 1.83\text{E}{-2}$	$\approx 3.59\text{E}{-7}$
	0.70	$\approx 8.80\%$	$\approx 13.2\%$	$\approx 6.92\%$	$n_{\text{index}} = 9$	$\approx 77.8\%$	$1 \times [7,1,7] \circ [255,131,37]$	131	2295	$\approx 1.90\text{E}{-2}$	$\approx 6.27\text{E}{-7}$
	0.74	$\approx 8.24\%$	$\approx 13.9\%$	$\approx 6.27\%$	$n_{\text{index}} = 11$	$\approx 81.8\%$	$1 \times [9,1,9] \circ [255,131,37]$	131	2805	$\approx 1.62\text{E}{-2}$	$\approx 5.72\text{E}{-8}$
	0.78	$\approx 7.57\%$	$\approx 14.6\%$	$\approx 5.58\%$	$n_{\text{index}} = 13$	$\approx 84.6\%$	$1 \times [11,1,11] \circ [255,131,37]$	131	3315	$\approx 1.65\text{E}{-2}$	$\approx 7.32\text{E}{-8}$
	0.82	$\approx 6.76\%$	$\approx 15.4\%$	$\approx 4.85\%$	$n_{\text{index}} = 16$	$\approx 68.8\%$	$1 \times [11,1,11] \circ [255,131,37]$	131	4080	$\approx 1.66\text{E}{-2}$	$\approx 7.57\text{E}{-8}$
	0.86	$\approx 5.80\%$	$\approx 16.4\%$	$\approx 4.07\%$	$n_{\text{index}} = 16$	$\approx 81.3\%$	$2 \times [13,1,13] \circ [255,71,59]$	142	8160	$\approx 3.57\text{E}{-2}$	$\approx 2.85\text{E}{-8}$
	0.90	$\approx 4.64\%$	$\approx 17.5\%$	$\approx 3.21\%$	$n_{\text{index}} = 16$	$\approx 81.3\%$	$3 \times [13,1,13] \circ [255,45,87]$	135	12240	$\approx 7.51\text{E}{-2}$	$\approx 6.42\text{E}{-7}$
von Neumann	0.50	$\approx 10.0\%$	$\approx 10.0\%$	$\approx 10.0\%$		$\approx 83.4\%$	$4 \times [8,1,8] \circ [63,36,11]$	144	2418	$\approx 2.73\text{E}{-3}$	$\approx 9.85\text{E}{-8}$
	0.54	$\approx 9.96\%$	$\approx 10.6\%$	$\approx 9.40\%$		$\approx 81.6\%$	$4 \times [8,1,8] \circ [63,36,11]$	144	2471	$\approx 2.72\text{E}{-3}$	$\approx 9.73\text{E}{-8}$
	0.58	$\approx 9.81\%$	$\approx 11.2\%$	$\approx 8.79\%$	3 passes	$\approx 77.0\%$	$4 \times [8,1,8] \circ [63,36,11]$	144	2617	$\approx 2.71\text{E}{-3}$	$\approx 9.37\text{E}{-8}$
	0.62	$\approx 9.58\%$	$\approx 11.8\%$	$\approx 8.18\%$		$\approx 70.7\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	2675	$\approx 8.70\text{E}{-4}$	$\approx 9.81\text{E}{-7}$
	0.66	$\approx 9.24\%$	$\approx 12.5\%$	$\approx 7.56\%$	multi-out	$\approx 63.6\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	2971	$\approx 8.52\text{E}{-4}$	$\approx 9.05\text{E}{-7}$
	0.70	$\approx 8.80\%$	$\approx 13.2\%$	$\approx 6.92\%$	$(n_2 \geq 8)$	$\approx 56.2\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	3365	$\approx 8.29\text{E}{-4}$	$\approx 8.12\text{E}{-7}$
	0.74	$\approx 8.24\%$	$\approx 13.9\%$	$\approx 6.27\%$		$\approx 48.6\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	3885	$\approx 8.00\text{E}{-4}$	$\approx 7.06\text{E}{-7}$
	0.78	$\approx 7.57\%$	$\approx 14.6\%$	$\approx 5.58\%$	retention	$\approx 41.4\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	4567	$\approx 7.65\text{E}{-4}$	$\approx 5.91\text{E}{-7}$
	0.82	$\approx 6.76\%$	$\approx 15.4\%$	$\approx 4.85\%$	yield 99%	$\approx 33.5\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	5650	$\approx 7.23\text{E}{-4}$	$\approx 4.72\text{E}{-7}$
	0.86	$\approx 5.80\%$	$\approx 16.4\%$	$\approx 4.07\%$		$\approx 26.1\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	7237	$\approx 6.73\text{E}{-4}$	$\approx 3.55\text{E}{-7}$
	0.90	$\approx 4.64\%$	$\approx 17.5\%$	$\approx 3.21\%$		$\approx 18.5\%$	$3 \times [10,1,10] \circ [63,45,7]$	135	10212	$\approx 6.13\text{E}{-4}$	$\approx 2.45\text{E}{-7}$

$\boldsymbol{p} \leftarrow \mathsf{SSGen}(\boldsymbol{x})$	$\widehat{\boldsymbol{y}} \leftarrow \mathsf{SSRep}(\widetilde{\boldsymbol{x}}, \boldsymbol{p})$	
Random $\boldsymbol{w} \in \mathcal{C}$ $\boldsymbol{p} \leftarrow \boldsymbol{x} \oplus \boldsymbol{w}$	$\widetilde{\boldsymbol{w}} \leftarrow \widetilde{\boldsymbol{x}} \oplus \boldsymbol{p} = \boldsymbol{w} \oplus \boldsymbol{e}$ $\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{w}} \leftarrow \mathsf{Correct}(\widetilde{\boldsymbol{w}})$	(a) Code-offset method of Juels et al. [21].
	$\widetilde{\boldsymbol{w}} \leftarrow \widetilde{\boldsymbol{x}} \oplus \boldsymbol{p} = \boldsymbol{w} \oplus \boldsymbol{e}$ $\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{x}} \leftarrow \boldsymbol{p} \oplus \mathsf{Correct}(\widetilde{\boldsymbol{w}})$	(b) Code-offset method of Dodis et al. [14].
	$\widetilde{\boldsymbol{w}} \leftarrow \widetilde{\boldsymbol{x}} \oplus \boldsymbol{p} = \boldsymbol{w} \oplus \boldsymbol{e}$ $\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{m}} \leftarrow \mathsf{Decode}(\widetilde{\boldsymbol{w}})$	(c) Code-offset method of Tuyls et al. [32].
$\boldsymbol{p} \leftarrow \boldsymbol{x} \cdot \boldsymbol{H}^T$	$\boldsymbol{s} \leftarrow \widetilde{\boldsymbol{x}} \cdot \boldsymbol{H}^T \oplus \boldsymbol{p} = \boldsymbol{e} \cdot \boldsymbol{H}^T$ Determine $\widehat{\boldsymbol{e}}$ $\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{x}} \leftarrow \widetilde{\boldsymbol{x}} \oplus \widehat{\boldsymbol{e}}$	(d) Syndrome method of Bennett et al. [5].
$\boldsymbol{p} \leftarrow \boldsymbol{x}(1:k) \cdot \boldsymbol{A}$ $\oplus\, \boldsymbol{x}(k+1:n)$	$\widehat{\boldsymbol{w}} \leftarrow \mathsf{Correct}(\widetilde{\boldsymbol{x}} \oplus (\boldsymbol{0} \Vert \boldsymbol{p}))$ $\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{x}} \leftarrow \widehat{\boldsymbol{w}} \oplus (\boldsymbol{0} \Vert \boldsymbol{p})$	(e) Systematic method of Yu [39].
	$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{x}}(1:k) \leftarrow \mathsf{Decode}(\widetilde{\boldsymbol{x}}$ $\oplus (\boldsymbol{0} \Vert \boldsymbol{p}))$	(f) Systematic method of Kang et al. [22].
$\boldsymbol{p} \leftarrow j$ so that $\boldsymbol{x} \in \mathcal{C}_j$	$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{m}} \leftarrow \mathsf{Decode}_{\mathcal{C}_j}(\widetilde{\boldsymbol{x}})$	(g) Multi-code method of Ahlswede et al. [1].



Tuyls et al. [32]
Ahlswede et al. [1]
[19]
Juels et al. [21]
Bennett et al. [5]
Kang et al. [22]
standard form
linear
Dodis et al. [14]
Yu [39]